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The Dynamic Equation on Hammer by Lagrange Equation in Robotic Arm

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Abstract: *- the dynamic equation of three freedoms is established hammer in robotic arm by Lagrange equation. It is based on one of two freedoms. It is found that the first and second items is long and others is concise which is found in calculated kinetic equation in this study. It compensates the blank in three freedoms for hammer of robotic arm by now. In the simulation occasion the kinetic equation will be computed in three freedoms according to this study.*

Keywords: *- Dynamic Equation, hammering, three freedoms; Robotic Arm, Lagrange Equation*

1 Introduction

In modern industry the activity of robotic arm is high as an automatic device. It is an important branch for the Robot. Its feature has been completing anticipated working task by program. It is an automatic device for robot technological field to be gained the most practical applications that is in industry manufacture, medical healing, military, semi-conductive manufacture and space exploring. Its structure and property have an advantage of human and robot respectively especial in exhibiting human intelligence and adaption. Its precision and capability in all kinds of environment is excellent so that it has wide prospects in each field of economy. The punching destruction is often used in destruction applications. It shall be studied detail that dynamic equation is established to grasp each parameters to wield its virtual use value. Because it has main two freedoms one is rotation and the other is punch it's this equation will be established according to these features. The each dynamic equation is plotted then substitute into Lagrange equation to search for the dynamic equation to be established. [1] Because it has three parts to move and hammering below equation is to be solved According to each parts movement. After establishing this equation it can be analyzed by related parameters to optimum and cost decreasing. The length and mass of components and position will be control parameters. [2] So in this paper these parameters will be further discussed to look for cost decreasing. It is hopeful to assist designer and related teacher in studying further at factory and university. In this paper the dynamic equation in the three freedoms of mechanical arms is established to satisfy the college research even industrial demand and further study in detail. In mechanical arm what the freedoms is chosen is most important to robot behavior because the three freedoms is more advanced and complicated than two freedoms. The amount of equation is important so it will be found that it is much or little to its complication to calculation course. Simplification and rapid and efficient computation is our aim.

2 Modeling and establishing dynamic equation

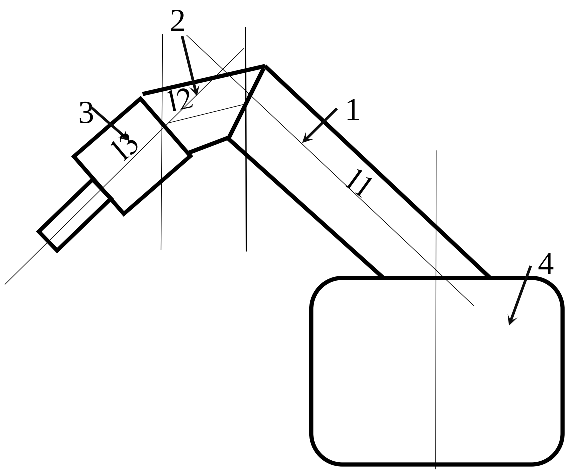

Figure 1 construction schematic of mechanical arm in series in robot

3-hand part; 2-wrist part; 1-arm part; 4-waist part.

In Figure 1 there are three freedoms in mechanical arm that name as 1~3. Meantime there are two other ones call 4&5.

System kinetic energy is

$$E_k = \frac{1}{2}\sum_{i}^{n}(m_1 v_1^2 + m_2 v_2^2 + m_3 v_3^2) \tag{1}$$

Here m_i: mass of i component ; J_{si}: rotary inertia of i component relative to center of mass; v_s: center of mass in i component; ω_i: angular velocity in i component; v_1, v_2 and v_3 is 1、2 and 3 velocities respectively.

$$v_D = \sqrt{\dot{X}_D^2 + \dot{Y}_D^2} \tag{2}$$

From Figure 2 it is known that position coordinate below

$$\begin{cases} X_D = \vec{l}_1 \sin\vartheta_1 + \vec{l}_2 \sin(\theta_1 + \theta_2) + \vec{l}_3 \sin(\theta_1 + \theta_3 + \theta_3) \\ Y_D = \vec{l}_1 \cos\theta_1 + \vec{l}_2 \cos(\theta_1 + \theta_2) + \vec{l}_3 \cos(\theta_1 + \theta_2 + \theta_3) \end{cases} \tag{3}$$

Derivating the equations we gain the $\dot{X}_C$, $\dot{Y}_C$ and $\dot{X}_3$ velocity in hand , $\dot{\theta}_1$, $\dot{\theta}_2$ and $\dot{\theta}_3$ one in joints. Suppose that the acceleration is $\ddot{\theta}_1$, $\ddot{\theta}_2$ and $\ddot{\theta}_3$ and the angular acceleration is $\ddot{\omega}_1$, $\ddot{\omega}_2$ and $\ddot{\omega}_3$ in joints.

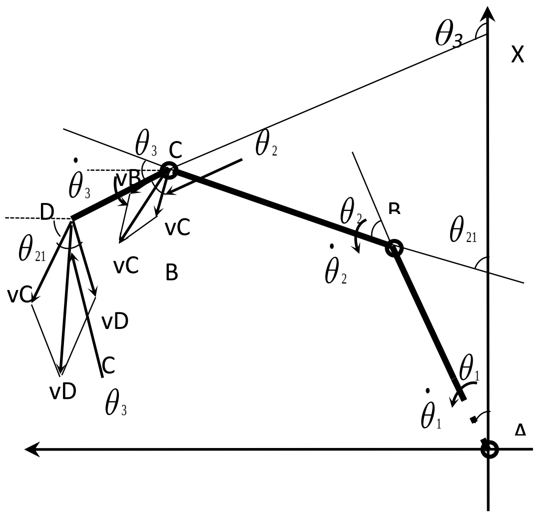

Figure 2 principle schematic of mechanical arm in series in robot

Here θ_{21} =360°-($\theta_1 + \theta_2$).

$$\begin{cases} \dot{X}_D = \dot{\theta}_1 \vec{l}_1 \cos\theta_1 + (\dot{\theta}_1 + \dot{\theta}_2)\vec{l}_2 \cos(\theta_1 + \theta_2) + (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)\vec{l}_3 \cos(\theta_1 + \theta_2 + \theta_3) \\ \dot{Y}_D = -\dot{\theta}_1 \vec{l}_1 \sin\theta_1 - (\dot{\theta}_1 + \dot{\theta}_2)\vec{l}_2 \sin(\theta_1 + \theta_2) - (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)\vec{l}_3 \sin(\theta_1 + \theta_2 + \theta_3) \end{cases} \tag{4}$$

v_B, v_C And v_D is B、C and D velocities respectively. So D point velocity is

$$v_D = \sqrt{\dot{X}_D^2 + \dot{Y}_D^2} = \sqrt{\begin{aligned} & l_1^2\theta_1^2 + l_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + l_3^2(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)^2 - 2l_1 l_2 \dot{\theta}_1(\dot{\theta}_1 + \dot{\theta}_2)\cos\theta_2 - 2l_1 l_3 \dot{\theta}_1 \\ & (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)\cos(\theta_1 + \theta_3) - 2l_2 l_3(\dot{\theta}_1 + \dot{\theta}_2)(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)\cos\theta_3 \end{aligned}} \tag{5}$$

C point velocity is

$$v_C = \sqrt{\dot{X}_C^2 + \dot{Y}_C^2} = \sqrt{l_1^2\theta_1^2 + l_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 - 2l_1 l_2 \dot{\theta}_1(\dot{\theta}_1 + \dot{\theta}_2)\cos\theta_2} \tag{6}$$

$$v_B = \vec{l}_1 \dot{\theta}_1 \tag{7}$$

Substituting two equations above to equation below

$$E_K = \frac{1}{2}\vec{l}_1^{\,2}(m_1+m_2+m_3)\theta_1^2 + \frac{1}{2}\vec{l}_2^{\,2} m_2(\theta_1+\theta_2)^2 + \frac{1}{2}\vec{l}_2^{\,2} m_2(\dot\theta_1+\dot\theta_2)^2 + \frac{1}{2}\vec{l}_3^{\,2} m_3 (\theta_1+\theta_2+\theta_3)^2 - 2\vec{l}_1\vec{l}_2 m_2\dot\theta_1(\dot\theta_1+\dot\theta_2)\cos\theta_2 - \vec{l}_1\vec{l}_2 m_3\dot\theta_1(\dot\theta_1+\dot\theta_2+\dot\theta_3)\cos(\theta_1+\theta_2) - \vec{l}_2\vec{l}_3 m_3(\dot\theta_1+\dot\theta_2)(\dot\theta_1+\dot\theta_2+\dot\theta_3)\cos\theta_3$$

(8)

Here

$$\frac{\partial E_K}{\partial\dot\theta_1} = (\dot\theta_1+\dot\theta_2)\vec{l}_2 m_2 - 2(\dot\theta_1+\dot\theta_2)\vec{l}_1\vec{l}_2 m_2\cos\theta_2 - 2\dot\theta_1\vec{l}_1\vec{l}_2 m_2\cos\theta_2 - \dot\theta_1\vec{l}_1\vec{l}_2 m_3 \cos(\theta_1+\theta_2) -_1 \vec{l}_2 m_3(\dot\theta_1+\dot\theta_2+\dot\theta_3)\cos(\theta_1+\theta_2) - \vec{l}_2 m_3(\dot\theta_1+\dot\theta_2)\cos\theta_3 - \vec{l}_2\vec{l}_3 m_3(\dot\theta_1+\dot\theta_2+\dot\theta_3)\cos\theta_3$$

((9)

$$\frac{\partial E_K}{\partial\dot\theta_2} = \vec{l}_2 m_2(\dot\theta_1+\dot\theta_2) - 2\vec{l}_1\vec{l}_2 m_2\dot\theta_1\cos\theta_2 - \vec{l}_1\vec{l}_2 m_2(\dot\theta_1+\dot\theta_2)\cos\theta_2 - \vec{l}_2\vec{l}_3 m_3(\dot\theta_1+\dot\theta_2+\dot\theta_2)\cos(\theta_1+\theta_2) - \vec{l}_2\vec{l}_3 m_3(\dot\theta_1+\dot\theta_2+\dot\theta_3)\cos\theta_3 - \vec{l}_2\vec{l}_3 m_3(\dot\theta_1+\dot\theta_2)\cos\theta_3$$

(10)

$$\frac{\partial E_K}{\partial\dot\theta_3} = -\vec{l}_1\vec{l}_2 m_3\dot\theta_1\cos(\theta_1+\theta_2+\theta_3) - \vec{l}_1\vec{l}_2 m_3\dot\theta_1\cos(\theta_1+\theta_2) - \vec{l}_2\vec{l}_3 m_3(\dot\theta_1+\dot\theta_3)\cos\theta_3$$

(11) And

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial\theta_1}\right) = \vec{l}_2 m_2(\ddot\theta_1+\ddot\theta_2) - 2(\ddot\theta_1+\ddot\theta_2)\vec{l}_1\vec{l}_2 m_2\cos\theta_1 + 2(\dot\theta_1+\dot\theta_2)\dot\theta_2\vec{l}_1\vec{l}_2 m_2\sin\theta_1 - 2\ddot\theta\vec{l}_1\vec{l}_2 m_2\cos\theta_2 + 2\dot\theta^2\vec{l}_1\vec{l}_2 m_2\sin\theta_2 - \ddot\theta\vec{l}_1\vec{l}_2 m_3\cos(\theta_1+\theta_2) + \dot\theta_1\vec{l}_1\vec{l}_2 m_3(\dot\theta_1+\dot\theta_2)\sin(\theta_1+\theta_2) - \vec{l}_2 m_3(\ddot\theta_1+\ddot\theta_2+\ddot\theta_3)\cos(\theta_1+\theta_2) - \vec{l}_2 m_3(\dot\theta_1+\dot\theta_2+\dot\theta_3)(\dot\theta_1+\dot\theta_2)\cos(\theta_1+\theta_2) - \vec{l}_2 m_3(\ddot\theta_1+\ddot\theta_2)\cos\theta_3 + \vec{l}_2 m_3(\dot\theta_1+\dot\theta_2)\dot\theta_3\cos\theta_3 - \vec{l}_1\vec{l}_2 m_3(\ddot\theta_1+\ddot\theta_2+\ddot\theta_3)\cos\theta_3 + \vec{l}_2\vec{l}_3 m_3(\ddot\theta_1+\ddot\theta_2+\ddot\theta_3)\cos\theta_3 + \vec{l}_2\vec{l}_3 m_3(\dot\theta_1+\dot\theta_2+\dot\theta_3)\dot\theta_3$$

(12)

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial\theta_2}\right) = -\vec{l}_2 m_2(\ddot\theta_2+\ddot\theta_1) - 2\vec{l}_1\vec{l}_2 m_2\ddot\theta_1\cos\theta_2 + 2\vec{l}_1\vec{l}_2 m_2\dot\theta_1\dot\theta_2\cos\theta_2 - \vec{l}_1\vec{l}_2 m_3(\ddot\theta_1+\ddot\theta_2)\cos(\theta_1+\theta_2) + \vec{l}_1\vec{l}_2 m_3(\dot\theta_1+\dot\theta_2+\dot\theta_3)(\dot\theta_1+\dot\theta_2)\sin(\theta_1+\theta_2) - \vec{l}_1\vec{l}_2 m_3(\ddot\theta_1+\ddot\theta_2+\ddot\theta_3)\cos\theta_1 + \vec{l}_1\vec{l}_2 m_3(\dot\theta_1+\dot\theta_2+\dot\theta_3)\dot\theta_1\sin\theta_1 - \vec{l}_2\vec{l}_3 m_3(\ddot\theta_1+\ddot\theta_2+\ddot\theta_3)\cos\theta_3 + \vec{l}_2\vec{l}_3 m_3(\dot\theta_1+\dot\theta_2+\dot\theta_3)\dot\theta_3\sin\theta_3$$

(13)

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial\theta_3}\right) = -\vec{l}_1\vec{l}_2 m_3\ddot\theta_1(\dot\theta_1+\dot\theta_2+\dot\theta_3) - \vec{l}_1\vec{l}_2 m_3\ddot\theta_1\cos(\theta_1+\theta_2) + \vec{l}_1\vec{l}_2 m_3\ddot\theta_1\cos(\theta_1+\theta_2) - \vec{l}_3\vec{l}_2 m_3(\ddot\theta_1+\ddot\theta_3)\cos\theta_3 + \vec{l}_2\vec{l}_3 m_3(\dot\theta_1+\dot\theta_3)\dot\theta_3\sin\theta_3$$

(14)

$$\frac{\partial E_K}{\partial\theta_1} = \vec{l}_1^{\,2}(m_1+m_2+m_3)\theta_1 + \vec{l}_2\vec{l}_3 m_2(\theta_1+\theta_2) + \vec{l}_3^{\,2} m_3(\theta_1+\theta_2+\theta_3) - \vec{l}_1\vec{l}_2 m_2\dot\theta_1(\dot\theta_1+\dot\theta_2+\dot\theta_3)\sin(\theta_1+\theta_2)$$

(15)

$$\frac{\partial E_K}{\partial\theta_2} = \vec{l}_2^{\,2} m_2(\theta_1+\theta_2) + \vec{l}_3^{\,2} m_3(\theta_1+\theta_2+\theta_3)$$

(16)

$$\frac{\partial E_K}{\partial\theta_3} = \vec{l}_3^{\,2}(\theta_1+\theta_2+\theta_3) - \vec{l}_2\vec{l}_3 m_3(\dot\theta_1+\dot\theta_2)(\dot\theta_1+\dot\theta_2+\dot\theta_3)\sin\theta_3$$

(17)

Potential energy of System

$$E_P = -\vec{l}_1 m_1 g\cos\theta_1 - \vec{l}_2 m_2 g\cos(\theta_1+\theta_2) - \vec{l}_3 m_3 g\cos(\theta_1+\theta_2+\theta_3)$$

(18)

$$\frac{\partial E_P}{\partial\theta_1} = \vec{l}_1 m_1 g\dot\theta_1\sin\theta_1 \qquad (19)$$

$$\frac{\partial E_P}{\partial\theta_2} = \vec{l}_2 m_2 g\dot\theta_2\sin(\theta_1+\theta_2)$$

$$\frac{\partial E_P}{\partial\theta_3} = \vec{l}_3 m_3 g\dot\theta_3\sin(\theta_1+\theta_2+\theta_3)$$

Substituting Lagrange equation below (20) for above. The equation of Lagrange equation is

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial\dot q_i}\right) - \frac{\partial E_K}{\partial q_i} + \frac{\partial E_P}{\partial q_i} = F_i, \quad (i=1,2,,...,n) \qquad (20)$$

Here E_K is kinetic of system;

E_P Is potential energy of system?

q_i Is generalized coordinate, it is a group of independent parameters that can define mechanical system movement;

F_i Is generalized force, when q_i is an angular displacement it a torque, when q_i is linear displacement it a force;

N is system generalized coordinate.

System generalized force is supposed that F_k(k=1,2,..,m) and M_j(j=1,2,..,n) is force and torque acting on system. Its power is

$$P=\sum_{k=1}^{m}(F_k v_k \cos\alpha_k)+\sum_{j=1}^{n}(\pm M_j \omega_j) \qquad (21)$$

Here ω_j: angular velocity acting on component with M_j;

v_k: the velocity in force F_k point of action; (the syntropy +, reverse direction -)

a_k: angle between F_k and v_k

When generalized coordinates is φ angular displacement generalized force=equivalent torque M_e.

$$\delta W_2=\sum_{k=1}^{m}(F_k \delta v_k \cos\alpha_k)+\sum_{j=1}^{n}(\pm M_j \delta\omega_j) \qquad (22)$$

$$\frac{\partial E_P}{\partial\theta_1}=\vec{l}_1\, m_1 g\, \dot{\theta}_1 \sin\theta_1$$

$$\frac{\partial E_P}{\partial\theta_2}=\vec{l}_2\, m_2 g\, \dot{\theta}_2 \sin(\theta_1+\theta_2)$$

$$\frac{\partial E_P}{\partial\theta_3}=\vec{l}_3\, m_3 g\, \dot{\theta}_3 \sin(\theta_1+\theta_2+\theta_3)$$

Here a_K is zero; F_K =; v_K =; ω_j = ; M_j =. $\delta\varphi_j$ Is virtual angular displacement; δs_k is virtual displacement. Supposing that

$$\delta s_K=\frac{\partial s_K}{\partial q_1}\delta q_1+\frac{\partial s_K}{\partial q_2}\delta q_2 \qquad (23)$$

$$\delta\varphi_K=\frac{\partial\varphi_j}{\partial q_1}\delta q_1+\frac{\partial\varphi_j}{\partial q_2}\delta q_2 \qquad (24)$$

Replace equation below with above two equations

$$\begin{cases} F_1=\sum_{k=1}^{m}\left[F_k\dfrac{\partial s_K}{\partial q_1}\cos\alpha_k\right]+\sum_{j=1}^{n}\left[M_J\dfrac{\partial\varphi_J}{\partial q_1}\right] \\ F_2=\sum_{k=1}^{m}\left[F_k\dfrac{\partial s_K}{\partial q_2}\cos\alpha_k\right]+\sum_{j=1}^{n}\left[M_J\dfrac{\partial\varphi_J}{\partial q_2}\right] \end{cases} \qquad (25)$$

This is generalized force equation.

3 Conclusions

In the modeling of three freedoms in hammer of robotic arm the kinetic equation is established according to Lagrange equation based on two freedoms robotic arm. It compensates the blank in three freedoms for robotic hammer arm. It is found that the first and second item is complicated and long besides others is concise. In the simulation occasion the kinetic equation will be computed on three freedoms according to this study.

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